AIEEE–2009, BOOKLET CODE(A)

Note: ***(i)*** The test is of 3 **hours** duration.

(ii) The test consists of **90** questions. The maximum marks are **432.**

(iii) There are ***three*** parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

Part A – Physics (144 marks)	–	***Question No. 1 to 2 and 9 to 30 consists FOUR (4) marks each and Question No. 3 to 8 consists EIGHT (8) marks each for each correct response.***
Part B – Chemistry (144 marks)	–	***Question No. 31 to 39 and 46 to 60 consists FOUR (4) marks each and Question No. 40 to 45 consists EIGHT (8) marks each for each correct response.***
Part C – Mathematics(144 marks)	–	***Question No. 61 to 82 and 89 to 90 consists FOUR (4) marks each and Question No. 83 to 88 consists EIGHT (8) marks each for each correct response.***

(iv) Candidates will be awarded marks as stated above for correct response of each question. $1/4^{th}$ marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

(v) * marked questions are from syllabus of class XI CBSE.

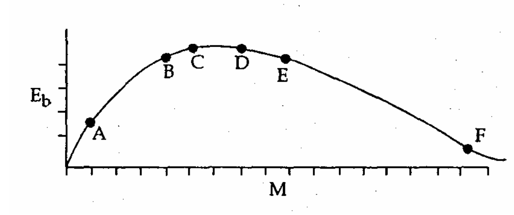
Physics

PART – A

1. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement – 1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.
Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero
(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(3) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true

Sol: **(2)**
Work done by conservative force does not depend on the path. Electrostatic force is a conservative force.

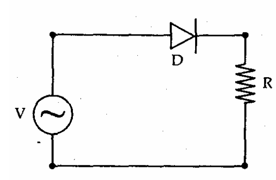
2. The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions:

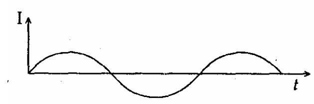


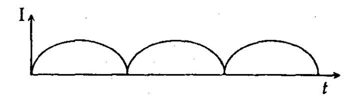
(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$
where ε is the energy released? In which reactions is ε positive?
(1) (i) and (iv) (2) (i) and (iii)
(3) (ii) and (iv) (4) (ii) and (iii)

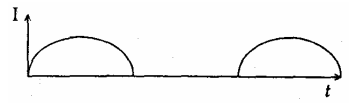
Sol: **(1)**
1st reaction is fusion and 4th reaction is fission.

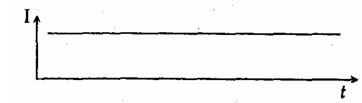
3. A p-n junction (D) shown in the figure can act as a rectifier. An alternating current source (V) is connected in the circuit.



(1) 

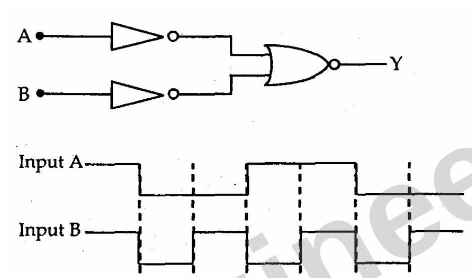
(2) 

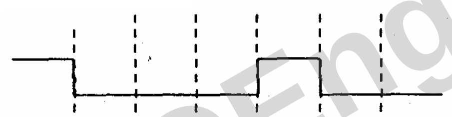
(3) 

(4) 

Sol: **(3)**
Given figure is half wave rectifier

4. The logic circuit shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform.



(1) 

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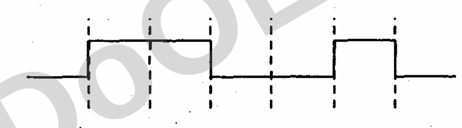
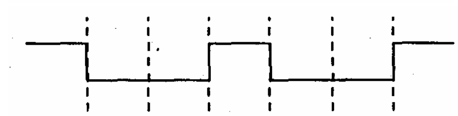
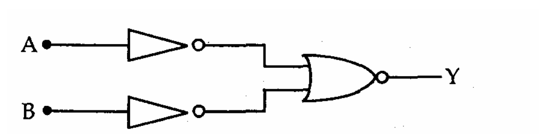
(3) 

(4) 

Sol: **(1)**

Truth Table

A	B	Y
1	1	1
1	0	0
0	1	0
0	0	0



*5. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

(1) $a^2T^2 + 4\pi^2v^2$

(2) $\frac{aT}{x}$

(3) $aT + 2\pi v$

(4) $\frac{aT}{v}$

Sol: **(2)**

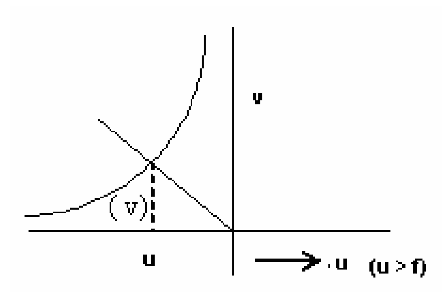
$$\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T^2}T = \frac{4\pi^2}{T} = \text{constant.}$$

6. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45^o with the x-axis meets the experimental curve at P. The coordinates of P will be

(1) (2f, 2f) (2) $\left(\frac{f}{2}, \frac{f}{2}\right)$

(3) (f, f) (4) (4f, 4f)

Sol: **(1)**
It is possible when object kept at centre of curvature.
u = v
u = 2f, v = 2f.



*7. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

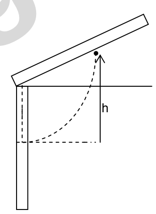
(1) $\frac{1}{3}\frac{\ell^2\omega^2}{g}$ (2) $\frac{1}{6}\frac{\ell\omega}{g}$

(3) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$ (4) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$

Sol: **(4)**
$T.E_i = T.E_f$

$\frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}\times\frac{1}{3}m\ell^2\omega^2 \quad mgh \Rightarrow h = \frac{1}{6}\frac{\ell^2\omega^2}{g}$



8. Let $P(r) = \frac{Q}{\pi R^4}r$ be the charge density distribution for a solid sphere of radius R and total charge Q. for a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

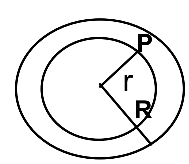
(1) 0 (2) $\frac{Q}{4\pi\varepsilon_o r_1^2}$

(3) $\frac{Qr_1^2}{4\pi\varepsilon_o R^4}$ (4) $\frac{Qr_1^2}{3\pi\varepsilon_o R^4}$

Sol: **(4)**

$\rho = \frac{Q}{\pi R^4}\times r_1$

$q_{in} = \frac{Q}{\pi R^4}r_1 \times \frac{4}{3} \quad r_1^3 = \frac{4}{3}\frac{Q}{R^4}r_1^4 \quad \pi$



By gauss law

$\oint E.dA = \frac{1}{\varepsilon_o}q_{in} = \frac{4}{3\varepsilon_o}\frac{Q}{R^4}r_1^4 \Rightarrow E\times 4\pi r_1^2 \quad \frac{4}{3\varepsilon_o}\frac{Q}{R^4}r_1^4 \Rightarrow E = \frac{Q}{3\pi\varepsilon_o R^4}r_1^2.$

9. The transition from the state n = 4 to n = 3 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from
(1) $2 \to 1$ (2) $3 \to 2$
(3) $4 \to 2$ (4) $5 \to 4$

Sol: (4)

IR corresponds to least value of $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

i.e. from Paschen, Bracket and Pfund series. Thus the transition corresponds to $5 \to 3$.

*10. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^{-3}. What is the energy of the gas due to its thermal motion?
(1) 3×10^4 J (2) 5×10^4 J
(3) 6×10^4 J (4) 7×10^4 J

Sol: (2)
Thermal energy corresponds to internal energy
Mass = 1 kg
density = 8 kg/m^3

$\Rightarrow$ Volume = $\frac{\text{mass}}{\text{density}} = \frac{1}{8}$m^3

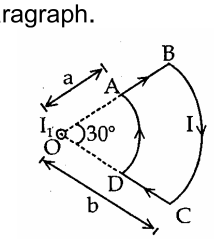
Pressure = 8×10^4 N/m^2

$\therefore$ Internal Energy = $\frac{5}{2}$ P $\times$ V = 5×10^4 J

11. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1: The temperature dependence of resistance is usually given as $R = R_o(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3} /^o C$.
Statement 2: $R = R_i (1 + \alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_o) << R_o$.
(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(3) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true

Sol: (1)

Directions: Question numbers 12 and 13 are based on the following paragraph.
A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



12. The magnitude of the magnetic field (B) due to loop ABCD at the origin (O) is

(1) zero

(2) $\frac{\mu_o (b-a)}{24ab}$

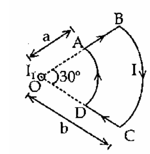
(3) $\frac{\mu_o I}{4\pi}\left[\frac{b-a}{ab}\right]$

(4) $\frac{\mu_o I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a \quad b)\right] +$

Sol: (2)

Net magnetic field due to loop ABCD at O is

$B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$



$= 0 + \frac{\mu_o I}{4\pi a} \times \frac{\pi}{6} \quad 0 \quad \frac{\mu_o I}{4\pi b} + \frac{\pi}{6} = \frac{\mu_o I}{24a} - \frac{\mu_o I}{24b} = \frac{\mu_o I}{24ab}(b-a)$

13. Due to the presence of the current I_1 at the origin

(1) The forces on AB and DC are zero

(2) The forces on AD and BC are zero

(3) The magnitude of the net force on the loop is given by $\frac{\mu_o I I_1}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a \quad b)\right] +$

(4) The magnitude of the net force on the loop is given by $\frac{\mu_o I I_1}{24ab}(b-a)$

Sol: (2)

The forces on AD and BC are zero because magnetic field due to a straight wire on AD and BC is parallel to elementary length of the loop.

14. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is

(1) 393.4 nm

(2) 885.0 nm

(3) 442.5 nm

(4) 776.8 nm

Sol: (3)

$3\lambda_1 = 4\lambda_2$

$\Rightarrow \lambda_2 = \frac{3}{4}\lambda_1 = \frac{3}{4} \times 590 = \frac{1770}{4}$ = 442.5 nm

15. Two points P and Q are maintained at the potentials of 10V and -4V respectively. The work done in moving 100 electrons from P to Q is

(1) -19×10^{-17} J

(2) 9.60×10^{-17} J

(3) -2.24×10^{-16} J

(4) 2.24×10^{-16} J

Sol: (4)

$W = QdV = Q(V_q - V_P) = -100 \times (1.6 \times 10^{-19}) \times (-4-10)$

$= +100 \times 1.6 \times 10^{-19} \times 14 = +2.24 \quad 10^{-16}$ J.

16. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV nm)

(1) 3.09 eV

(2) 1.41 eV

(3) 151 eV

(4) 1.68 eV

Sol: (2)

$\frac{1}{2}mv^2 = eV_o = 1.68eV \Rightarrow h\nu = \frac{hc}{\lambda} = \frac{1240evnm}{400nm} = 3.1\,eV \Rightarrow 3.1\,eV = W_o + 1.6\,eV$

$\therefore \quad W_o = 1.42$ eV

*17. A particle has an initial velocity $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is

(1) 10 units (2) $7\sqrt{2}$ units
(3) 7 units (4) 8.5 units

Sol: **(2)**

$\vec{u} = 3\hat{i} + 4\hat{j}$; $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$

$\vec{u} = \vec{u} + \vec{a}t$

$= 3\hat{i} + 4\hat{j} + \left(0.4\hat{i} + 0.3\hat{j}\right) \times 10 = 3\hat{i} + \hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$

Speed is $\sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

*18. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric sire. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (speed of sound = 330 ms^{-1}).

(1) 49 m (2) 98 m
(3) 147 m (4) 196 m

Sol: **(2)**

Motor cycle, u = 0, a = 2 m/s^2

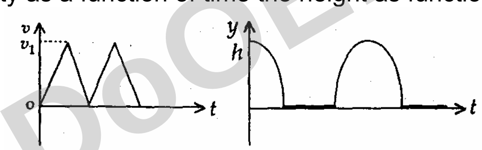
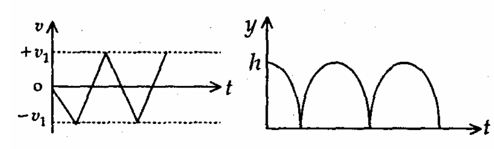
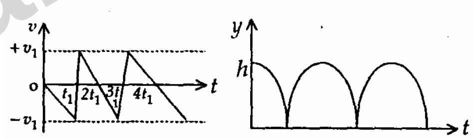
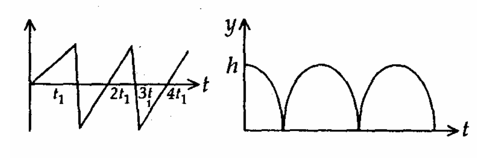
Observer is in motion and source is at rest.

$\Rightarrow n' = n\dfrac{v - v_O}{v + v_S} \Rightarrow \dfrac{94}{100}n = n\dfrac{330 - v_O}{330} \Rightarrow 330 - v_O = \dfrac{330 \times 94}{100}$

$\Rightarrow v_O = 330 - \dfrac{94 \times 33}{10} = \dfrac{33 \times 6}{10}$ m/s

s = $\dfrac{v^2 - u^2}{2a} = \dfrac{9 \times 33 \times 33}{100} = \dfrac{9 \times 1089}{100} \approx 98$ m.

*19. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time the height as function of time will be

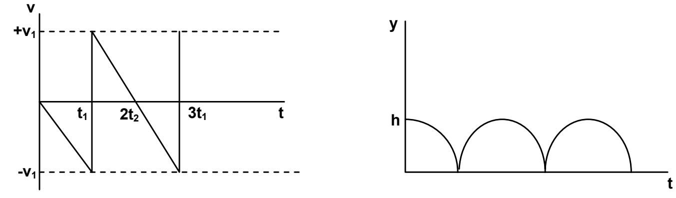
(1) 
(2) 
(3) 
(4) 

Sol: **(3)**

$h = \dfrac{1}{2}gt^2$, v = - gt and after the collision, v = gt.

(parabolic) (straight line)

Collision is perfectly elastic then ball reaches to same height again and again with same velocity.



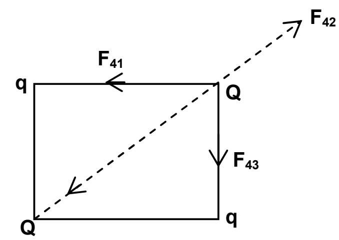
20. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the Q/q equals

(1) $-2\sqrt{2}$ (2) -1

(3) 1 (4) $-\dfrac{1}{\sqrt{2}}$

Sol: **(1)**

Three forces F_{41}, F_{42} and f_{43} acting on Q are shown



Resultant of $F_{41} + F_{43}$

$= \sqrt{2}\ F_{each}$

$= \sqrt{2}\dfrac{1}{4\pi\varepsilon_o}\dfrac{Qq}{d^2}$

Resultant on Q becomes zero only when 'q' charges are of negative nature.

$$F_{4,2} = \frac{1}{4\pi\varepsilon_o}\frac{Q\times Q}{\left(\sqrt{2}d\right)^2}$$

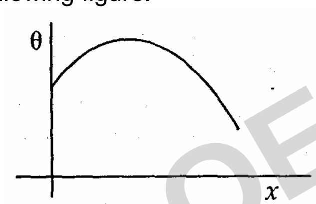
$$\Rightarrow \sqrt{2}\frac{dQ}{d^2} = \frac{Q\times Q}{2d^2}$$

$$\Rightarrow \sqrt{2}\times q = \frac{Q\times Q}{2}$$

$$\therefore\ q = -\frac{Q}{2\sqrt{2}} \text{ or } \frac{Q}{q} = -2\sqrt{2}$$

*21. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figure.

(1)



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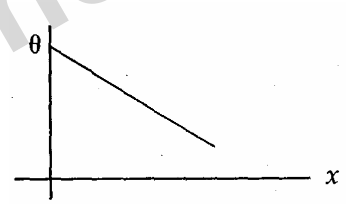

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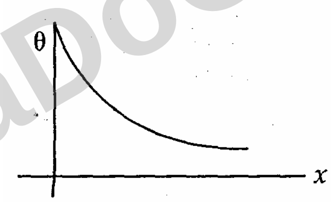

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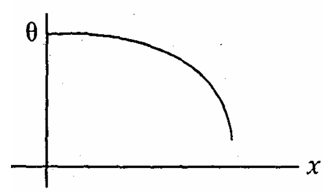


Sol: **(2)**

We know that $\dfrac{dQ}{dt} = KA\dfrac{d\theta}{dx}$

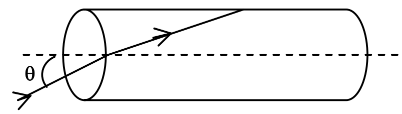
In steady state flow of heat

$$d\theta = \frac{dQ}{dt}.\frac{1}{kA}.dx$$

$\Rightarrow \theta_H - \theta = k'x \Rightarrow \theta = \theta_H - k'x$

Equation $\theta = \theta_H - k'x$ represents a straight line.

22. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid point of one end of the rod as shown in the figure.

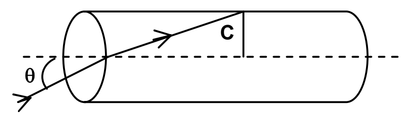


The incident angle θ for which the light ray grazes along the wall of the rod is

(1) $\sin^{-1}\left(\frac{1}{2}\right)$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(3) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Sol: (4)



$\text{Sin}C = \frac{\sqrt{3}}{2}$ ..... (1)

$\text{Sin } r = \sin(90 - C) = \cos C = \frac{1}{2}$

$\frac{\sin\theta}{\sin r} = \frac{\mu_2}{\mu_1}$

$\sin\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

*23. Three sound waves of equal amplitudes have frequencies (v – 1), v, (v + 1). They *superpose* to give beats. The number of beats produced per second will be

(1) 4 (2) 3
(3) 2 (4) 1

Sol: (3)

Maximum number of beats = $\nu + 1 - (\nu - 1) = 2$

*24. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is

(1) 2R (2) $\frac{R}{\sqrt{2}}$

(3) $\frac{R}{2}$ (4) $\sqrt{2}\,R$

Sol: (1)

$g' = \frac{GM}{(R+h)^2}$, acceleration due to gravity at height h

$\Rightarrow \frac{g}{9} = \frac{GM}{R^2}.\frac{R^2}{(R+h)^2} = g\left(\frac{R}{R+h}\right)^2$

$\Rightarrow \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$

$\Rightarrow$ 3R = R + h $\Rightarrow$ 2R = h

*25. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire-2 has cross-sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(1) F (2) 4F
(3) 6F (4) 9F

Sol: (4)

$$A_1\ell_1 = A_2\ell_2 \Rightarrow \ell_2 = \frac{A_1\ell_1}{A_2} = \frac{A\times\ell_1}{3A} \quad \frac{1}{3} \Rightarrow \frac{\ell_1}{\ell_2} = 3 \quad \ell$$

$$\Delta x_1 = \frac{F_1}{A\gamma}\times\ell_1 \quad \text{..... (i)}$$

$$\Delta x_2 = \frac{F_2}{3A\gamma}\ell_2 \quad \text{.....(ii)}$$

Here $\Delta x_1 = \Delta x_2$

$$\frac{F_2}{3A\gamma}\ell_2 = \frac{F_1}{A\gamma}\ell_1$$

$$F_2 = 3F_1\times\frac{\ell_1}{\ell_2} = 3F_1 \quad 3 = 9F\times$$

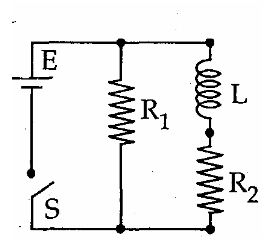
*26. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree(=0.5^o), then the least count of the instrument is

(1) one minute (2) half minute
(3) one degree (4) half degree

Sol: (1)

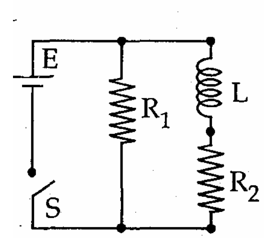
$$\text{Least count} = \frac{\text{value of main scale division}}{\text{No of divisions on vernier scale}} = \frac{1}{30}\text{MSD} = \frac{1}{30}\times\frac{1^o}{2} = \frac{1}{60}^o = 1 \text{ minute}$$

27. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2Ω and R_2 = 2Ω are connected to a battery of emf 12V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is



(1) $6e^{-5t}V$ (2) $\frac{12}{t}e^{-3t}V$
(3) $6\left(1-e^{-t/0.2}\right)V$ (4) $12e^{-5t}V$

Sol: (4)



$$I_1 = \frac{F}{R_1} = \frac{12}{2} = 6A$$

$$E = L\frac{dI_2}{dt} + R_2 \quad I_2 \qquad \times$$

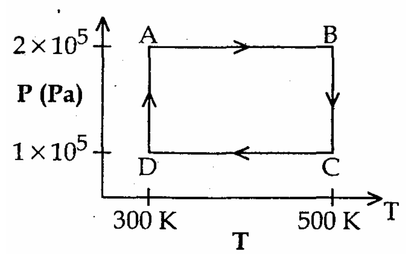
$$I_2 = I_o\left(1-e^{-t/t_c}\right) \Rightarrow I_o = \frac{E}{R_2} = \frac{12}{2} \quad 6A \qquad =$$

$$t_c = \frac{L}{R} = \frac{400\times10^{-3}}{2} \quad 0.2 \; =$$

$$I_2 = 6\left(1-e^{-t/0.2}\right)$$

Potential drop across L = $E - R_2I_2 = 12 - 2\times 6\,(1 - e^{-bt}) = 12\,e^{-5t}$

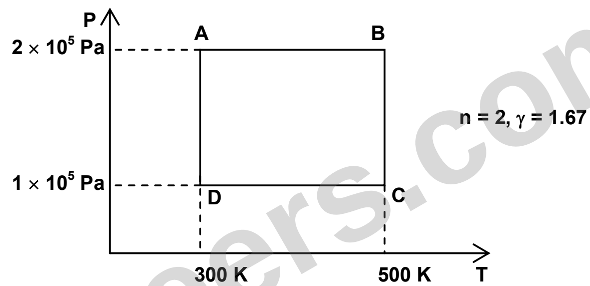
Directions: Question numbers 28, 29 and 30 are based on the following paragraph.
Two moles of helium gas are taken over the cycle ABCDA, as shown in the P – T diagram.



*28. Assuming the gas to be ideal the work done on the gas in taking it from A to B is
(1) 200 R (2) 300 R
(3) 400 R (4) 500 R

Sol: **(3)**

$W_{AB} = \Delta Q - \Delta U = nC_pdT - nC_vdT$ (at constant pressure)
$= n(C_p - C_v)dt$
$= nRdT = 2 \times R \times (500 - 300) = 400$ R



*29. The work done on the gas in taking it from D to A is
(1) – 414 R (2) + 414 R
(3) – 690 R (4) + 690 R

Sol: **(1)**
At constant temperature (isothermal process)

$$W_{DA} = nRT\ln\left(\frac{P_1}{P_2}\right) = 2.303\times2R\times300\log\left(\frac{10^5}{2\times10^5}\right)$$
$$= 2.303\times600R\log\left(\frac{1}{2}\right)$$
$$= 0.693 \times 600 \text{ R} = -414 \text{ R}.$$

*30. The net work done on the gas in the cycle ABCDA is
(1) Zero (2) 276 R
(3) 1076 R (4) 1904 R

Sol: **(2)**

Net work done in a cycle = $W_{AB} + W_{Bc} + W_{CB} + W_{BA}$
$= 400\text{ R} + 2 \times 2.303 \times 500\text{ R}\ln 2 - 400\text{R} - 414\text{ R}$
$= 1000\text{R} \times \ln 2 - 600\text{R} \times \ln 2 = 400\text{R} \times \ln 2 = 276\text{R}$

CHEMISTRY

PART – B

31. Knowing that the Chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements in incorrect ?
(1) Because of the large size of the Ln (III) ions the bonding in its compounds is predominantly ionic in character.
(2) The ionic sizes of Ln (III) decrease in general with increasing atomic number.
(3) Ln (III) compounds are generally colourless.
(4) Ln (III) hydroxides are mainly basic in character.

Sol: (3)
Ln^{+3} compounds are mostly coloured.

32. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was :
(1) CH_3OH (2) HCHO
(3) CH_3COCH_3 (4) CH_3COOH

Sol: (4)
Esterification reaction is involved

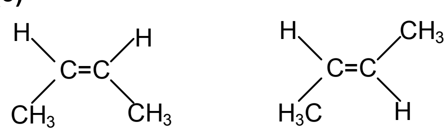
$$CH_3COOH_{(\ell)} + C_2H_5OH_{(\ell)} \xrightarrow{H^+} CH_3COOC_2H_{5(\ell)} + H_2O_{(\ell)}$$

*33. Arrange the carbanions, $(CH_3)_3\bar{C}$, $\bar{C}Cl_3$, $(CH_3)_2\bar{C}H$, $C_6H_5\bar{C}H_2$, in order of their decreasing stability :
(1) $C_6H_5\bar{C}H_2 > \bar{C}Cl_3 > (CH_3)_3\bar{C} > (CH_3)_2\bar{C}H$ (2) $(CH_3)_2\bar{C}H > \bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_3\bar{C}$
(3) $\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$ (4) $(CH_3)_3\bar{C} > (CH_3)_2\bar{C}H > C_6H_5\bar{C}H_2 > \bar{C}Cl_3$

Sol: (3)
2^o carbanion is more stable than 3^o and Cl is –I effect group.

*34. The alkene that exhibits geometrical isomerism is :
(1) propene (2) 2-methyl propene
(3) 2-butene (4) 2- methyl -2- butene

Sol: (3)



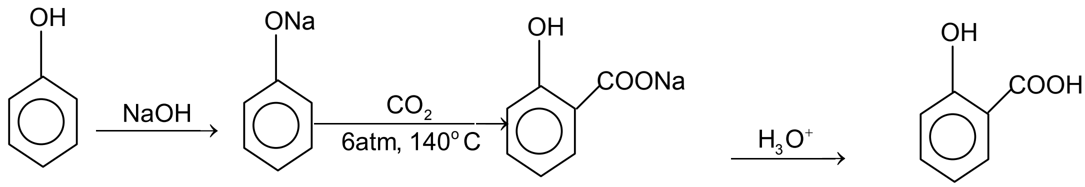
cis Trans

*35. In which of the following arrangements, the sequence is not strictly according to the property written against it ?
(1) $CO_2 < SiO_2 < SnO_2 < PbO_2$: increasing oxidising power
(2) HF< HCl < HBr < HI : increasing acid strength
(3) $NH_3 < PH_3 < AsH_3 < SbH_3$: increasing basic strength
(4) B < C < O < N : increasing first ionization enthalpy.

Sol: (3)
Correct basic strength is $NH_3 > PH_3 > AsH_3 > BiH_3$

36. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is :
(1) benzoic acid
(2) salicylaldehyde
(3) salicylic acid
(4) phthalic acid

Sol: **(3)**
Kolbe – Schmidt reaction is



Salicylic Acid

37. Which of the following statements is incorrect regarding physissorptions ?
(1) It occurs because of vander Waal's forces.
(2) More easily liquefiable gases are adsorbed readily.
(3) Under high pressure it results into multi molecular layer on adsorbent surface.
(4) Enthalpy of adsorption $(\Delta H_{adsorption})$ is low and positive.

Sol: **(4)**
Enthalpy of adsorption regarding physissorption is not positive and it is negative.

38. Which of the following on heating with aqueous KOH, produces acetaldehyde ?
(1) CH_3COCl
(2) CH_3CH_2Cl
(3) $CH_2Cl\ CH_2Cl$
(4) CH_3CHCl_2

Sol: **(4)**

$$CH_3CHCl_2 \xrightarrow{aq.KOH} CH_3CH(OH)_2 \xrightarrow[-H_2O]{} CH_3CHO$$

*39. In an atom, an electron is moving with a speed of 600m/s with an accuracy of 0.005%. Certainity with which the position of the electron can be located is $(h = 6.6 \times 10^{-34}\ kg\ m^2s^{-1}$, mass of electron, $e_m = 9.1 \times 10^{-31} kg)$
(1) $1.52 \times 10^{-4} m$
(2) $5.10 \times 10^{-3} m$
(3) $1.92 \times 10^{-3} m$
(4) $3.84 \times 10^{-3} m$

Sol: **(3)**

$$\Delta x.m\ \Delta v = \frac{h}{4\pi}$$

$$\Delta x = \frac{h}{4\pi\ m\Delta v}$$

$$\Delta v = 600 \times \frac{0.005}{100} = 0.03$$

$$\Rightarrow \Delta x = \frac{6.625 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03} = 1.92 \times 10^{-3} m$$

40. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is $CH_3OH(\ell)+\frac{3}{2}O_2(g)\rightarrow CO_2(g)\ \ 2H_2O(\ell)$ At 298K standard Gibb's energies of formation for $CH_3OH(\ell), H_2O(\ell)$ and $CO_2(g)$ are -166.2, -237.2 and -394.4 kJ mol^{-1} respectively. If standard enthalpy of combustion of methanol is -726kJ mol^{-1}, efficiency of the fuel cell will be

(1) 80 %
(2) 87%
(3) 90%
(4) 97%

Sol: **(4)**

$CH_3OH(\ell)+\frac{3}{2}O_2(g)\rightarrow CO_2(g)+2H_2O(\ell)\ \ \Delta H=\ \ 726kJ\ mol^{-1}$

Also $\Delta G_f^o CH_3OH(\ell)$ = -166.2 kJ mol^{-1}

$\Delta G_f^o H_2O(\ell)$ = -237.2 kJ mol^{-1}

$\Delta G_f^o CO_2(\ell)$ = -394.4 kJ mol^{-1}

$\because$ $\Delta G = \Sigma\Delta G_f^o$ products $-\Sigma\Delta G_f^o$ reactants.

= -394.4 -2 (237.2) + 166.2

= -702.6 kJ mol^{-1}

now Efficiency of fuel cell = $\frac{\Delta G}{\Delta H}\times 100$

= $\frac{702.6}{726}\times 100$

= 97%

41. Two liquids X and Y form an ideal solution. At 300K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mm Hg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 mm Hg. Vapour pressure (in mmHg) of X and Y in their pure states will be, respectively :

(1) 200 and 300
(2) 300 and 400
(3) 400 and 600
(4) 500 and 600

Sol: **(3)**

$P_T = P_X^o x_X + P_Y^o x_Y$

x_X = mol fraction of X

x_Y = mol fraction of Y

$\therefore$ 550 = $P_x^o\left(\frac{1}{1+3}\right)+P_Y^o\ \frac{3}{1+3}\ \ \left(\quad\right)$

= $\frac{P_X^o}{4}+\frac{3P_Y^o}{4}$

$\therefore$ 550 (4) = $P_X^o+3P_Y^o$ .... (1)

Further 1 mol of Y is added and total pressure increases by 10 mm Hg.

$\therefore$ 550 + 10 = $P_X^o\left(\frac{1}{1+4}\right)+P_Y^o\ \frac{4}{1+4}\ \ \left(\quad\right)$

$\therefore$ 560 (5) = $P_X^o+4P_Y^o$ ....(2)

By solving (1) and (2)

We get, $P_X^o = 400$ mm Hg

P_Y^o = 600 mm Hg

42. The half life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be (log 2=0.301) :
(1) 230.3 minutes (2) 23.03 minutes
(3) 46.06 minutes (4) 460.6 minutes

Sol: (3)

$\because \lambda = \frac{0.6932}{t_{1/2}} = \frac{0.6932}{6.93} min^{-1}$

Also $t = \frac{2.303}{\lambda} \log \frac{[A_o]}{[A]}$

$[A_o]$ = initial concentration (amount)

$[A]$ = final concentration (amount)

$\therefore t = \frac{2.303 \times 6.93}{0.6932} \log \frac{100}{1}$

= 46.06 minutes

43. Given : $E^o_{Fe^{3+}/Fe} = -0.036V$, $E^o_{Fe^{2+}/Fe}$ = -0.439V. The value of standard electrode potential for the change, $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2}$ (aq) will be :
(1) -0.072 V (2) 0.385 V
(3) 0.770 V (4) -0.270

Sol: (3)

$\because Fe^{3+} + 3e^- \rightarrow Fe; E^o = -0.036V$

$\therefore \Delta G^O_1 = -nFE^o = -3F(-0.036)$

= +0.108 F

Also $Fe^{2+} + 2e^- \rightarrow Fe;\ E^o = -0.439$ V

$\therefore \Delta G^O_2$ = -nF E^o

= -2 F(-0.439)

= 0.878 F

To find E^o for $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}$ (aq)

ΔG^O = -nFE^o

= -1FE^o

$\because G^o = G^o_1 - G^o_2$

$\therefore G^o$ = 0.108F - 0.878F

$\therefore$ -FE^o = +0.108F – 0.878F

$\therefore E^O$ = 0.878 - 0.108

= 0.77v

*44. On the basis of the following thermochemical data : ($\Delta fG^oH^+_{(aq)} = 0$)

$H_2O(\ell) \rightarrow H^+(aq) + OH^-(aq);\ \Delta H = 57.32kJ$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(\ell)\ ;\ \Delta H = -286.20kJ$

The value of enthalpy of formation of OH^- ion at 25^oC is :
(1) -22.88 kJ (2) -228.88 kJ
(3) +228.88 kJ (4) -343.52 kJ

Sol: (2)

By adding the two given equations, we have

$H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H^+_{(aq)} + OH^-_{(aq)}$; ΔH =-228.88 Kj

Here ΔH^o_f of $H^+_{(aq)} = 0$

$\therefore \Delta H^o_f$ of OH^- = -228.88 kJ

45. Copper crystallizes in fcc with a unit cell length of 361 pm. What is the radius of copper atom ?
(1) 108 pm
(2) 127 pm
(3) 157 pm
(4) 181 pm

Sol: **(2)**
For FCC,
$\sqrt{2}a = 4r$ (the atoms touches each other along the face- diagonal)
r = $\frac{\sqrt{2}a}{4} = \frac{\sqrt{2} \times 361}{4}$
= 127 pm

46. Which of the following has an optical isomer ?
(1) $\left[CO(NH_3)_3\,Cl\right]^{+}$
(2) $\left[CO(en)\,NH_3\,(_2\right]^{2+}$)
(3) $\left[CO\,(H_2O)_4\quad en\right]^{3+}$ ( )
(4) $\left[CO(en)_2\,\,NH_3\,(_2\right]^{3+}$)

Sol: **(4)**
It is an octahedral complex of the type $\left[M(AA)_2\,X_2\right]$
Where AA is bidentate ligand.

*47. Solid Ba $(NO_3)_2$ is gradually dissolved in a $1.0 \times 10^{-4}M$ Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form ?(K_{sp} for Ba CO_3 = 5.1 × 10^{-9}).
(1) 4.1 $\times 10^{-5}M$
(2) $5.1 \times 10^{-5}M$
(3) $8.1 \times 10^{-8}M$
(4) $8.1 \times 10^{-7}M$

Sol: **(2)**
$Ba(NO_3)_2 + CaCO_3 \rightarrow BaCO_3 \quad 2NaNO_3 \quad +$
Here $\left[CO_3^{-2}\right] = \left[Na_2CO_3\right] = 10^{-4}M$
$K_{sp} = \left[Ba^{+2}\right]\,CO_3^{-2} \Rightarrow 5.1 \times 10^{-9} = \left[Ba^{2+}\right](10^{-4}) \quad Ba^{2} \Rightarrow 5.1 \times 10^{-5}$] =
At this value, just precipitation starts.

48. Which one of the following reactions of Xenon compounds is not feasible ?
(1) $XeO_3 + 6HF \rightarrow Xe\,F_6 + 3H_2O$
(2) $3Xe\,F_4 + 6H_2O \rightarrow 2\,Xe \quad XeO_3 \quad 12\,HF + 1.5\,O_2 \quad +$
(3) $2XeF_2 + 2H_2O \rightarrow 2Xe \quad 4HF \quad O_2 \quad + \quad +$
(4) $XeF_6 + RbF \rightarrow Rb(XeF_7]$

Sol: **(1)**
Remaining are feasible

*49. Using MO theory predict which of the following species has the shortest bond length ?
(1) O_2^{2+}
(2) O_2^{+}
(3) O_2^{-}
(4) O_2^{2-}

Sol: **(1)**
Bond length $\alpha \frac{1}{\text{bond order}}$
Bond order = $\frac{\text{no..of bonding } \bar{e} - \text{no.of antibonding } \bar{e}}{2}$
Bond orders of O_2^{+}, O_2^{-}, O_2^{-2} and O_2^{+2} are respectively
2.5, 1.5, 1 and 3.

50. In context with the transition elements, which of the following statements is incorrect ?
(1) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements

in complexes.
(2) In the highest oxidation states, the transition metal show basic character and form cationic complexes.
(3) In the highest oxidation states of the first five transition elements (Sc to Mn), all the 4s and 3d electrons are used for bonding.
(4) Once the d^5 configuration is exceeded, the tendency to involve all the 3d electrons in bonding decreases.

Sol: **(2)**
In higher Oxidation states transition elements show acidic nature

*51. Calculate the wavelength (in nanometer) associated with a proton moving at $1.0 \times 10^3\,ms^{-1}$
(Mass of proton = 1.67 $\times 10^{-27}$ kg and $h = 6.63 \times 10^{-34}$ Js) :
(1) 0.032 nm (2) 0.40 nm
(3) 2.5 nm (4) 14.0 nm

Sol: **(2)**
$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^3} \equiv 0.40\,nm$$

52. A binary liquid solution is prepared by mixing n-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution ?
(1) The solution formed is an ideal solution
(2) The solution is non-ideal, showing +ve deviation from Raoult's law.
(3) The solution is non-ideal, showing –ve deviation from Raoult's law.
(4) n-heptane shows +ve deviation while ethanol shows –ve deviation from Raoult's law.

Sol: **(2)**
The interactions between n –heptane and ethanol are weaker than that in pure components.

*53. The number of stereoisomers possible for a compound of the molecular formula $CH_3 - CH = CH - CH(OH)$ Me is : –
(1) 3 (2) 2
(3) 4 (4) 6

Sol: **(3)**
About the double bond, two geometrical isomers are possible and the compound is having one chiral carbon.

*54. The IUPAC name of neopentane is
(1) 2-methylbutane (2) 2, 2-dimethylpropane
(3) 2-methylpropane (4) 2,2-dimethylbutane

Sol: **(2)**

```
            CH3
            |
Neopentane is H3C – C – CH3
            |
            CH3
```

*55. The set representing the correct order of ionic radius is :
(1) $Li^+ > Be^{2+} > Na$ Mg^2 $^+>$ $^+$ (2) $Na^+ > Li^+ > Mg^{2+}$ Be^2 $>$ $^+$
(3) $Li^+ > Na^+ > Mg^2$ Be^2 $^+>$ $^+$ (4) $Mg^{2+} > Be^{2+} > Li^+$ Na $>$ $^+$

Sol: **(2)**
Follow the periodic trends
56. The two functional groups present in a typical carbohydrate are :
(1) -OH and -COOH (2) -CHO and -COOH
(3) > C = O and - OH (4) - OH and -CHO

Sol: **(3)**
Carbohydrates are polyhydroxy carbonyl compounds.

*57. The bond dissociation energy of B – F in BF_3 is 646 kJ mol^{-1} whereas that of C-F in CF_4 is 515kJ mol^{-1}. The correct reason for higher B-F bond dissociation energy as compared to that of C- F is :
(1) smaller size of B-atom as compared to that of C- atom
(2) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4
(3) significant $p\pi$- $p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4.
(4) lower degree of $p\pi$ - $p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4.

Sol: **(3)**
option itself is the reason

58. In Cannizzaro reaction given below

$$2\text{ Ph CHO} \xrightarrow{:\overset{(-)}{OH}} \text{Ph } CH_2OH + PhC\ddot{O}_2^{(-)}$$ the slowest step is :

(1) the attack of $:\overset{(-)}{OH}$ at the carboxyl group
(2) the transfer of hydride to the carbonyl group
(3) the abstraction of proton from the carboxylic group
(4) the deprotonation of Ph CH_2OH

Sol: **(2)**
Hydride transfer is the slowest step.

59. Which of the following pairs represents linkage isomers ?
(1) $[Cu(NH_3)_4][Pt\ Cl_4]$ and $[Pt(NH_3)_4][CuCl_4]$
(2) $[Pd(P\ Ph_3)_2\ NCS_2]$ and $[Pd\ (P\ Ph_3)_2\ (SCN)_2]$
(3) $[CO\ (NH_3)_5\ NO_3]SO_4$ and $[CO(NH_3)_5\ SO_4]NO_3$
(4) $[PtCl_2\ (NH_3)_4]Br_2$ and $[PtBr_2\ (NH_3)_4]Cl_2$

Sol: **(2)**
NCS^- is ambidentate ligand and it can be linked through N (or) S

60. Buna-N synthetic rubber is a copolymer of :

(1) $H_2C = CH - \underset{}{\overset{Cl}{\overset{|}{C}}} = CH_2$ and $H_2C = CH - CH = CH_2$
(2) $H_2C = CH - CH = CH_2$ and $H_5C_6 - CH = CH_2$
(3) $H_2C = CH - CN$ and $H_2C = CH - CH = CH_2$
(4) $H_2C = CH - CN$ and $H_2C = CH - \underset{CH_3}{\underset{|}{C}} = CH_2$

Sol: **(3)**

Mathematics

PART – C

61. Let a, b, c be such that $b(a+c)\neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$, then the value of 'n' is

(1) zero
(2) any even integer
(3) any odd integer
(4) any integer

Sol: **(3)**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n}\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix} = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n}\begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - (-1)^{n+1}\begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix} = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2}\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

This is equal to zero only if n + 2 is odd i.e. n is odd integer.

62. If the mean deviation of number 1, 1 + d, 1 + 2d, ….. , 1 + 100d from their mean is 255, then the d is equal to

(1) 10.0
(2) 20.0
(3) 10.1
(4) 20.2

Sol: **(3)**

$$\text{Mean}(\bar{x}) = \frac{\text{sum of quantities}}{n} = \frac{\frac{n}{2}(a+l)}{n} = \frac{1}{2}[1+1+100d] = 1+50d$$

$$\text{M.D.} = \frac{1}{n}\sum|x_i-\bar{x}| \Rightarrow 255 = \frac{1}{101}[50d+49d+48d+....+d+0+d+.....+50d] = \frac{2d}{101}\left[\frac{50 \times 51}{2}\right]$$

$$\Rightarrow d = \frac{255 \times 101}{50 \times 51} = 10.1$$

*63. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is

(1) greater than 4ab
(2) less than 4ab
(3) greater than – 4ab
(4) less than – 4ab

Sol: **(3)**

$bx^2+cx+a=0$

Roots are imaginary $\Rightarrow c^2-4ab<0 \Rightarrow c^2<4ab \Rightarrow -c^2>-4ab$

$3b^2x^2+6bcx+2c^2$

since $3b^2>0$

Given expression has minimum value

Minimum value = $\dfrac{4(3b^2)(2c^2)-36b^2c^2}{4(3b^2)} = -\dfrac{12b^2c^2}{12b^2} = -c^2 > -4ab$.

*64. Let A and B denote the statements

A: $\cos\alpha + \cos\beta + \cos\gamma = 0$

B: $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

(1) A is true and B is false
(2) A is false and B is true
(3) both A and B are true
(4) both A and B are false

Sol: **(3)**

$$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$$

$$\Rightarrow 2\left[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)\right] + 3 = 0$$

$$\Rightarrow 2\left[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)\right] + \sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\gamma = 0$$

$$\Rightarrow (\sin\alpha + \sin\beta + \sin\gamma)^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$$

*65. The lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2 x + (p^2+1)y + 2q = 0$ are perpendicular to a common line for

(1) no value of p
(2) exactly one value of p
(3) exactly two values of p
(4) more than two values of p

Sol: **(2)**

Lines must be parallel, therefore slopes are equal $\Rightarrow p(p^2+1) = -(p^2+1) \Rightarrow p = -1$

66. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(1) A = B
(2) A = C
(3) B = C
(4) $A \cap B = \phi$

Sol: **(3)**

67. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $\left[3\vec{u}\ \ p\vec{v}\ \ p\vec{w}\right] - \left[p\vec{v}\ \ \vec{w}\ \ q\vec{u}\right] - \left[2\vec{w}\ \ q\vec{v}\ \ q\vec{u}\right] = 0$ holds for

(1) exactly one value of (p, q)
(2) exactly two values of (p, q)
(3) more than two but not all values of (p , q)
(4) all values of (p, q)

Sol: **(1)**

$$(3p^2 - pq + 2q^2)\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right] = 0$$

But $\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right] \neq 0$

$$3p^2 - pq + 2q^2 = 0$$

$$2p^2 + p^2 - pq + \left(\frac{q}{2}\right)^2 + \frac{7q^2}{4} = 0 \Rightarrow 2p^2 + \left(p - \frac{q}{2}\right)^2 + \frac{7}{4}q^2 = 0$$

$\Rightarrow p = 0, q = 0, p = \frac{q}{2}$

This possible only when p = 0, q = 0 exactly one value of (p, q)

68. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals

(1) (6, - 17)
(2) (- 6, 7)
(3) (5, - 15)
(4) (- 5, 15)

Sol: **(2)**

Dr's of line = $(3, -5, 2)$

Dr's of normal to the plane = $(1, 3, -\alpha)$

Line is perpendicular to normal $\Rightarrow 3(1) - 5(3) + 2(-\alpha) = 0 \Rightarrow 3 - 15 - 2\alpha = 0 \Rightarrow -2\alpha = 12 \Rightarrow \alpha = -6$

Also $(2, 1, -2)$ lies on the plane

$2 + 3 + 6(-2) + \beta = 0 \Rightarrow \beta = 7$

$\therefore (\alpha, \beta) = (-6, 7)$

*69. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(1) less than 500
(2) at least 500 but less than 750
(3) at least 750 but less than 1000
(4) at least 1000

Sol: **(4)**

4 novels can be selected from 6 novels in ${}^6C_4$ ways. 1 dictionary can be selected from 3 dictionaries in ${}^3C_1$ ways. As the dictionary selected is fixed in the middle, the remaining 4 novels can be arranged in 4! ways.

$\therefore$ The required number of ways of arrangement = ${}^6C_4 \times {}^3C_1 \times 4! = 1080$

70. $\int_0^{\pi} [\cot x]\, dx$, $[\bullet]$ denotes the greatest integer function, is equal to

(1) $\frac{\pi}{2}$
(2) 1
(3) – 1
(4) $-\frac{\pi}{2}$

Sol: **(4)**

Let $I = \int_0^{\pi} [\cot x]\, dx$ ...(1)

$= \int_0^{\pi} [\cot(\pi - x)]\, dx = \int_0^{\pi} [-\cot x]\, dx$ ...(2)

Adding (1) and (2)

$2I = \int_0^{\pi} [\cot x]\, dx + \int_0^{\pi} [-\cot x]\, dx = \int_0^{\pi} (-1)\, dx$

$\left[\because [x] + [-x] = -1 \text{ if } x \notin Z \right.$
$\left. \quad = 0 \text{ if } x \in Z \right]$

$= [-x]_0^{\pi} = -\pi$

$\therefore I = -\frac{\pi}{2}$

71. For real x, let $f(x) = x^3 + 5x + 1$, then

(1) f is one-one but not onto R
(2) f is onto R but not one-one
(3) f is one-one and onto R
(4) f is neither one-one nor onto R

Sol: **(3)**

Given $f(x) = x^3 + 5x + 1$

Now $f'(x) = 3x^2 + 5 > 0, \ \forall x \in R$

$\therefore$ f(x) is strictly increasing function

$\therefore$ It is one-one

Clearly, f(x) is a continuous function and also increasing on R,

$\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

$\therefore$ f(x) takes every value between $-\infty$ and ∞.

Thus, f(x) is onto function.

72. In a binomial distribution $B\left(n,\ p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

(1) $\frac{1}{\log_{10}^{4}-\log_{10}^{3}}$ (2) $\frac{1}{\log_{10}^{4}+\log_{10}^{3}}$

(3) $\frac{9}{\log_{10}^{4}-\log_{10}^{3}}$ (4) $\frac{4}{\log_{10}^{4}-\log_{10}^{3}}$

Sol: **(1)**

$1-q^n \ge \frac{9}{10}$ $\left(\frac{3}{4}\right)^n \le \frac{1}{10}$ $n \le \log_{\frac{3}{4}} 10 \Rightarrow n \ge \frac{1}{\log_{10}^{4}-\log_{10}^{3}}$

*73. If P and Q are the points of intersection of the circles $x^2+y^2+3x+7y+2p-5=0$ and $x^2+y^2+2x+2y-p^2=0$, then there is a circle passing through P, Q and (1, 1) for

(1) all values of p (2) all except one value of p

(3) all except two values of p (4) exactly one value of p

Sol: **(1)**

Given circles $S = x^2+y^2+3x+7y+2p-5=0$

$S' = x^2+y^2+2x+2y-p^2=0$

Equation of required circle is $S+\lambda S'=0$

As it passes through (1, 1) the value of λ= -(7+2p)/(6-p^2)

If 7 + 2p = 0, it becomes the second circle

$\therefore$ it is true for all values of p

74. The projections of a vector on the three coordinate axis are 6, - 3, 2 respectively. The direction cosines of the vector are

(1) $6, -3, 2$ (2) $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$

(3) $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$ (4) $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

Sol: **(3)**

Projection of a vector on coordinate axis are $x_2-x_1, y_2-y_1, z_2-z_1$

$x_2-x_1=6,\ y_2-y_1=-3,\ z_2-z_1=2$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}=\sqrt{36+9+4}=7$

The D.C's of the vector are $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

*75. If $\left|Z-\frac{4}{z}\right|=2$, then the maximum value of $|Z|$ is equal to

(1) $\sqrt{3}+1$ (2) $\sqrt{5}+1$

(3) 2 (4) $2+\sqrt{2}$

Sol: **(2)**

$|Z|=\left|\left(Z-\frac{4}{Z}\right)+\frac{4}{Z}\right| \Rightarrow |Z| \le \left|Z-\frac{4}{Z}\right|+\left|\frac{4}{Z}\right|$

$\Rightarrow |Z| \le \left|Z-\frac{4}{Z}\right|+\frac{4}{|Z|} \Rightarrow |Z| \le 2+\frac{4}{|Z|}$

$\Rightarrow |Z|^2 - 2|Z| - 4 \le 0$

$\left(|Z| - \left(\sqrt{5}+1\right)\right)\left(|Z| - \left(1-\sqrt{5}\right)\right) \le 0 \Rightarrow 1-\sqrt{5} \le |Z| \le \sqrt{5}+1$

*76. Three distinct points A, B and C are given in the 2 – dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (- 1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point

(1) $(0, 0)$
(2) $\left(\frac{5}{4}, 0\right)$
(3) $\left(\frac{5}{2}, 0\right)$
(4) $\left(\frac{5}{3}, 0\right)$

Sol: **(3)**

$P = (1, 0)$; $Q(-1, 0)$

Let $A = (x, y)$

$\frac{AP}{AQ} = \frac{BP}{BQ} = \frac{CP}{CQ} = \frac{1}{3}$ ..(1)

$\Rightarrow 3AP = AQ \Rightarrow 9AP^2 = AQ^2 \Rightarrow 9(x-1)^2 + 9y^2 = (x+1)^2 + y^2$

$\Rightarrow 9x^2 - 18x + 9 + 9y^2 = x^2 + 2x + 1 + y^2 \Rightarrow 8x^2 - 20x + 8y^2 + 8 = 0$

$\Rightarrow x^2 + y^2 - 5x + 1 = 0$ ...(2)

$\therefore$ A lies on the circle

Similarly B, C are also lies on the same circle

$\therefore$ Circumcentre of ABC = Centre of Cricle (1) = $\left(\frac{5}{2}, 0\right)$

*77. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

(1) 0
(2) 2
(3) 7
(4) 8

Sol: **(2)**

$8^{2n} - (62)^{2n+1} = (1+63)^n - (63-1)^{2n+1}$

$= \left(1 + {}^nc_1 63 + {}^nc_2 63^2 + ... + 63^n\right) - \left(-1 + {}^{(2n+1)}c_1 63 - {}^{2n+1}c_2 (63)^2 + ... + 63^{(2n+1)}\right)$

$= 2 + 63\left({}^nc_1 + {}^nc_2 (63) + ... + 63^{n-1} - {}^{(2n+1)}c_1 + {}^{2n+1}c_2 (63) - .... - 63^{(2n)}\right)$

$\therefore$ Reminder is 2

*78. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn in inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is

(1) $x^2 + 16y^2 = 16$
(2) $x^2 + 12y^2 = 16$
(3) $4x^2 + 48y^2 = 48$
(4) $4x^2 + 64y^2 = 48$

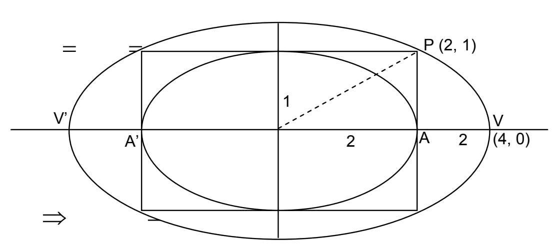
Sol: **(2)**

$x^2 + 4y^2 = 4 \Rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1 \Rightarrow a = 2, b = 1 \Rightarrow P(2, 1)$

Required Ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$

(2, 1) lies on it

$\Rightarrow \frac{4}{16} + \frac{1}{b^2} = 1 \Rightarrow \frac{1}{b^2} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow b^2 = \frac{4}{3}$



$$\therefore \frac{x^2}{16}+\frac{y^2}{\left(\frac{4}{3}\right)}=1 \Rightarrow \frac{x^2}{16}+\frac{3y^2}{4}=1 \Rightarrow x^2+12y^2=16$$

*79. The sum to the infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$ is

(1) 2
(2) 3
(3) 4
(4) 6

Sol: **(2)**

Let $S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$ …(1)

$\frac{1}{3}S=\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+\ldots$ …(2)

Dividing (1) & (2)

$S\left(1-\frac{1}{3}\right)=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\ldots$

$$\frac{2}{3}S=\frac{4}{3}+\frac{4}{3^2}\left(1+\frac{1}{3}+\frac{1}{3^2}+\ldots\right)\Rightarrow \frac{2}{3}S=\frac{4}{3}+\frac{4}{3^2}\left(\frac{1}{1-\frac{1}{3}}\right)=\frac{4}{3}+\frac{4}{3^2}\cdot\frac{3}{2}=\frac{4}{3}+\frac{2}{3}=\frac{6}{2}\Rightarrow\frac{2}{3}S=\frac{6}{3}\Rightarrow S=3$$

80. The differential equation which represents the family of curves $y=c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants is

(1) $y'=y^2$
(2) $y''=y'y$
(3) $yy''=y'$
(4) $yy''=(y')^2$

Sol: **(4)**

$y=c_1e^{c_2x}$ …(1)

$y'=c_2c_1e^{c_2x}$

$y'=c_2y$ …(2)

$y''=c_2y'$

From (2)

$c_2=\frac{y'}{y}$

So, $y''=\frac{(y')^2}{y}\Rightarrow yy''=(y')^2$

81. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

(1) $\frac{1}{14}$
(2) $\frac{1}{7}$
(3) $\frac{5}{14}$
(4) $\frac{1}{50}$

Sol: **(1)**

S = { 00, 01, 02, …., 49 }

Let A be the even that sum of the digits on the selected ticket is 8 then

A = { 08, 17, 26, 35, 44 }

Let B be the event that the product of the digits is zero

B = { 00, 01, 02, 03, …., 09, 10, 20, 30, 40 }

$A\cap B=\{8\}$

Required probability = $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{1}{50}}{\frac{14}{50}} = \dfrac{1}{14}$

82. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(1) – 1 (2) 1

(3) log 2 (4) – log 2

Sol: **(1)**

$x^{2x} - 2x^x \cot y - 1 = 0 \quad \ldots(1)$

Now x = 1,

1 – 2 coty – 1 = 0 ⇒ coty = 0 ⇒ y = $\dfrac{\pi}{2}$

Now differentiating eq. (1) w.r.t. ‘x’

$$2x^{2x}(1+\log x) - 2\left[x^x(-\operatorname{cosec}^2 y)\frac{dy}{dx} + \cot y\, x^x(1+\log x)\right] = 0$$

Now at $\left(1, \dfrac{\pi}{2}\right)$

$$2(1+\log 1) - 2\left(1 \cdot 1 \cdot (-1)\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} + (0)(1+0)\right) = 0$$

$$\Rightarrow 2 + 2\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = -1$$

83. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent to the parabola at the point (2, 3) and the x-axis is

(1) 3 (2) 6

(3) 9 (4) 12

Sol: **(3)**

Equation of tangent at (2, 3) to $(y-2)^2 = x - 1$ is $S_1 = 0$

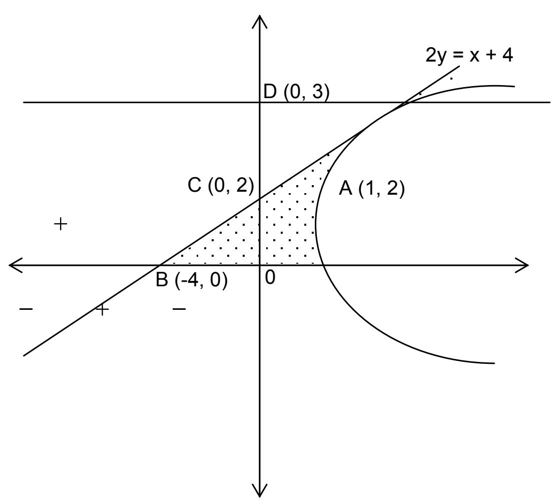
$\Rightarrow x - 2y + 4 = 0$

Required Area = Area of ΔOCB + Area of OAPD – Area of ΔPCD

$= \dfrac{1}{2}(4 \times 2) + \int_0^3 (y^2 - 4y + 5)\,dy - \dfrac{1}{2}(1 \times 2)$

$= 4 + \left[\dfrac{y^3}{3} - 2y^2 + 5y\right]_0^3 - 1 = 4 + 9 - 18 + 15 - 1$

$= 28 - 19 = 9$ sq. units



(or)

Area = $\int_0^3 \left((y-2)^2 + 1 - (2y-4)\right) dy = \int_0^3 (y^2 - 6y + 9)\,dy = \int_0^3 (3-y)^2\,dy = \left[-\dfrac{(y-3)^3}{3}\right]_0^3 = \dfrac{27}{3} = 9$ sq.units

84. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$

(1) $P(-1)$ is the minimum and $P(1)$ is the maximum of P

(2) $P(-1)$ is not minimum but $P(1)$ is the maximum of P

(3) $P(-1)$ is the minimum and $P(1)$ is not the maximum of P

(4) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P

Sol: **(2)**

$P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$\because$ x = 0 is a solution for $P'(x) = 0$, $\Rightarrow$ c = 0

$\therefore P(x) = x^4 + ax^3 + bx^2 + d$ …(1)

Also, we have $P(-1) < P(1)$

$\Rightarrow 1 - a + b + d < 1 + a + b + d \Rightarrow a > 0$

$\because P'(x) = 0$, only when x = 0 and P(x) is differentiable in (- 1, 1), we should have the maximum and minimum at the points x = - 1, 0 and 1 only

Also, we have $P(-1) < P(1)$

$\therefore$ Max. of P(x) = Max. { P(0), P(1) } & Min. of P(x) = Min. { P(-1), P(0) }

In the interval [0 , 1],

$P'(x) = 4x^3 + 3ax^2 + 2bx = x(4x^2 + 3ax + 2b)$

$\because P'(x)$ has only one root x = 0, $4x^2 + 3ax + 2b = 0$ has no real roots.

$\therefore (3a)^2 - 32b < 0 \Rightarrow \frac{3a^2}{32} < b$

$\therefore$ b > 0

Thus, we have a > 0 and b > 0

$\therefore P'(x) = 4x^3 + 3ax^2 + 2bx > 0, \forall x \in (0, 1)$

Hence P(x) is increasing in [0, 1]

$\therefore$ Max. of P(x) = P(1)

Similarly, P(x) is decreasing in [-1 , 0]

Therefore Min. P(x) does not occur at x = - 1

85. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{2\sqrt{3}}{8}$

(3) $\frac{3\sqrt{2}}{5}$ (4) $\frac{\sqrt{3}}{4}$

Sol: **(1)**

$x - y + 1 = 0$ …(1)

$x = y^2$

$1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$ = Slope of given line (1)

$\frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2} \Rightarrow y = \frac{1}{2} \Rightarrow x = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \Rightarrow (x, y) = \left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ The shortest distance is $\frac{\left|\frac{1}{4} - \frac{1}{2} + 1\right|}{\sqrt{1+1}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

Directions: Question number 86 to 90 are Assertion – Reason type questions. Each of these questions contains two statements

Statement-1 (Assertion) and Statement-2 (Reason).
Each of these questions also have four alternative choices, only one of which is the correct answer. You have to select the correct choice

86. Let $f(x) = (x+1)^2 - 1, x \ge -1$
Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$
Statement-2 : f is a bijection.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

Sol: **(3)**
There is no information about co-domain therefore f(x) is not necessarily onto.

87. Let $f(x) = x|x|$ and $g(x) = \sin x$.
Statement-1 : gof is differentiable at x = 0 and its derivative is continuous at that point.
Statement-2 : gof is twice differentiable at x = 0.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

Sol: **(3)**
$f(x) = x|x|$ and $g(x) = \sin x$

$$gof(x) = \sin(x|x|) = \begin{cases} -\sin x^2 & , x < 0 \\ \sin x^2 & , x \ge 0 \end{cases}$$

$$\therefore (gof)'(x) = \begin{cases} -2x\cos x^2 & , x < 0 \\ 2x\cos x^2 & , x \ge 0 \end{cases}$$

Clearly, $L(gof)'(0) = 0 = R(gof)'(0)$
$\therefore$ gof is differentiable at x = 0 and also its derivative is continuous at x = 0

$$\text{Now, } (gof)''(x) = \begin{cases} -2\cos x^2 + 4x^2 \sin x^2 & , x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2 & , x \ge 0 \end{cases}$$

$\therefore L(gof)''(0) = -2$ and $R(gof)''(0) = 2$
$\therefore L(gof)''(0) \ne R(gof)''(0)$
$\therefore$ gof(x) is not twice differentiable at x = 0.

*88. Statement-1 : The variance of first n even natural numbers is $\frac{n^2 - 1}{4}$

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

Sol: **(4)**
Statement-2 is true

Statement-1:
Sum of n even natural numbers = n (n + 1)

$$\text{Mean}(\bar{x}) = \frac{n(n+1)}{n} = n + 1$$

$$\text{Variance} = \left[\frac{1}{n}\sum (x_i)^2\right] - (\bar{x})^2 = \frac{1}{n}\left[2^2 + 4^2 + \ldots\ldots + (2n)^2\right] - (n+1)^2$$

$$= \frac{1}{n}2^2\left[1^2 + 2^2 + \ldots\ldots + n^2\right] - (n+1)^2 = \frac{4}{n}\frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{(n+1)\left[2(2n+1) - 3(n+1)\right]}{3} = \frac{(n+1)\left[4n+2-3n-3\right]}{3} = \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$$

$\therefore$ Statement 1 is false.

89. Statement-1 : $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement-2 : $\sim(p \leftrightarrow \sim q)$ is a tautology.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

Sol: **(3)**

p	q	$p \leftrightarrow q$	~q	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	T	T	F	T

90. Let A be a 2 x 2 matrix

Statement-1 : $\text{adj}(\text{adj } A) = A$

Statement-2 : $|\text{adj } A| = |A|$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

Sol: **(2)**

$$|\text{adj } A| = |A|^{n-1} = |A|^{2-1} = |A|$$

$$\text{adj}(\text{adj } A) = |A|^{n-2} A = |A|^0 A = A$$

AIEEE–2009, ANSWER KEY

Test Booklet Code-A			Test Booklet Code-B			Test Booklet Code-C			Test Booklet Code-D		
PHY	CHE	MAT	CHE	MAT	PHY	MAT	PHY	CHE	CHE	PHY	MAT
1. **(2)**	31. **(3)**	61. **(3)**	1. **(1)**	31. **(2)**	61. **(2)**	1. **(4)**	31. **(4)**	61. **(4)**	1. **(1)**	31. **(4)**	61. **(4)**
2. **(1)**	32. **(4)**	62. **(3)**	2. **(4)**	32. **(1)**	62. **(2)**	2. **(4)**	32. **(2)**	62. **(3)**	2. **(1)**	32. **(3)**	62. **(4)**
3. **(3)**	33. **(3)**	63. **(3)**	3. **(1)**	33. **(2)**	63. **(1)**	3. **(4)**	33. **(3)**	63. **(4)**	3. **(1)**	33. **(4)**	63. **(1)**
4. **(1)**	34. **(3)**	64. **(3)**	4. **(3)**	34. **(3)**	64. **(3)**	4. **(3)**	34. **(4)**	64. **(1)**	4. **(1)**	34. **(4)**	64. **(1)**
5. **(2)**	35. **(3)**	65. **(2)**	5. **(3)**	35. **(2)**	65. **(1)**	5. **(2)**	35. **(2)**	65. **(3)**	5. **(4)**	35. **(1)**	65. **(2)**
6. **(1)**	36. **(3)**	66. **(3)**	6. **(1)**	36. **(2)**	66. **(1)**	6. **(1)**	36. **(3)**	66. **(4)**	6. **(2)**	36. **(3)**	66. **(3)**
7. **(4)**	37. **(4)**	67. **(1)**	7. **(1)**	37. **(1)**	67. **(2)**	7. **(4)**	37. **(1)**	67. **(3)**	7. **(1)**	37. **(3)**	67. **(2)**
8. **(3)**	38. **(4)**	68. **(2)**	8. **(1)**	38. **(4)**	68. **(4)**	8. **(4)**	38. **(4)**	68. **(4)**	8. **(1)**	38. **(4)**	68. **(4)**
9. **(4)**	39. **(3)**	69. **(4)**	9. **(2)**	39. **(1)**	69. **(1)**	9. **(4)**	39. **(2)**	69. **(4)**	9. **(1)**	39. **(3)**	69. **(4)**
10. **(2)**	40. **(4)**	70. **(4)**	10. **(1)**	40. **(3)**	70. **(4)**	10. **(3)**	40. **(2)**	70. **(4)**	10. **(4)**	40. **(1)**	70. **(2)**
11. **(1)**	41. **(3)**	71. **(3)**	11. **(2)**	41. **(4)**	71. **(4)**	11. **(1)**	41. **(1)**	71. **(3)**	11. **(4)**	41. **(4)**	71. **(3)**
12. **(2)**	42. **(3)**	72. **(1)**	12. **(4)**	42. **(1)**	72. **(1)**	12. **(3)**	42. **(4)**	72. **(4)**	12. **(4)**	42. **(4)**	72. **(3)**
13. **(2)**	43. **(3)**	73. **(1)**	13. **(3)**	43. **(2)**	73. **(3)**	13. **(4)**	43. **(2)**	73. **(1)**	13. **(4)**	43. **(2)**	73. **(1)**
14. **(3)**	44. **(2)**	74. **(3)**	14. **(3)**	44. **(2)**	74. **(4)**	14. **(3)**	44. **(1)**	74. **(4)**	14. **(1)**	44. **(4)**	74. **(4)**
15. **(4)**	45. **(2)**	75. **(2)**	15. **(2)**	45. **(3)**	75. **(3)**	15. **(2)**	45. **(4)**	75. **(4)**	15. **(3)**	45. **(2)**	75. **(4)**
16. **(2)**	46. **(4)**	76. **(3)**	16. **(2)**	46. **(2)**	76. **(2)**	16. **(4)**	46. **(3)**	76. **(3)**	16. **(3)**	46. **(2)**	76. **(1)**
17. **(2)**	47. **(2)**	77. **(2)**	17. **(2)**	47. **(1)**	77. **(1)**	17. **(3)**	47. **(1)**	77. **(4)**	17. **(1)**	47. **(2)**	77. **(2)**
18. **(2)**	48. **(1)**	78. **(2)**	18. **(1)**	48. **(4)**	78. **(1)**	18. **(3)**	48. **(2)**	78. **(4)**	18. **(1)**	48. **(2)**	78. **(1)**
19. **(3)**	49. **(1)**	79. **(2)**	19. **(1)**	49. **(2)**	79. **(2)**	19. **(4)**	49. **(3)**	79. **(3)**	19. **(1)**	49. **(1)**	79. **(1)**
20. **(1)**	50. **(2)**	80. **(4)**	20. **(2)**	50. **(1)**	80. **(1)**	20. **(1)**	50. **(3)**	80. **(2)**	20. **(4)**	50. **(1)**	80. **(3)**
21. **(2)**	51. **(2)**	81. **(1)**	21. **(2)**	51. **(2)**	81. **(4)**	21. **(2)**	51. **(3)**	81. **(2)**	21. **(4)**	51. **(3)**	81. **(1)**
22. **(4)**	52. **(2)**	82. **(1)**	22. **(2)**	52. **(2)**	82. **(3)**	22. **(3)**	52. **(4)**	82. **(3)**	22. **(1)**	52. **(3)**	82. **(4)**
23. **(3)**	53. **(3)**	83. **(3)**	23. **(2)**	53. **(4)**	83. **(3)**	23. **(3)**	53. **(1)**	83. **(3)**	23. **(2)**	53. **(1)**	83. **(4)**
24. **(1)**	54. **(2)**	84. **(2)**	24. **(1)**	54. **(2)**	84. **(1)**	24. **(3)**	54. **(3)**	84. **(4)**	24. **(4)**	54. **(3)**	84. **(1)**
25. **(4)**	55. **(2)**	85. **(1)**	25. **(1)**	55. **(2)**	85. **(4)**	25. **(4)**	55. **(1)**	85. **(1)**	25. **(2)**	55. **(4)**	85. **(1)**
26. **(1)**	56. **(3)**	86. **(3)**	26. **(1)**	56. **(1)**	86. **(1)**	26. **(1)**	56. **(2)**	86. **(1)**	26. **(4)**	56. **(4)**	86. **(3)**
27. **(4)**	57. **(3)**	87. **(3)**	27. **(3)**	57. **(1)**	87. **(3)**	27. **(2)**	57. **(1)**	87. **(2)**	27. **(2)**	57. **(1)**	87. **(1)**
28. **(3)**	58. **(2)**	88. **(4)**	28. **(2)**	58. **(4)**	88. **(4)**	28. **(4)**	58. **(3)**	88. **(3)**	28. **(1)**	58. **(2)**	88. **(3)**
29. **(1)**	59. **(2)**	89. **(3)**	29. **(2)**	59. **(4)**	89. **(2)**	29. **(2)**	59. **(3)**	89. **(4)**	29. **(2)**	59. **(3)**	89. **(1)**
30. **(2)**	60. **(3)**	90. **(2)**	30. **(2)**	60. **(3)**	90. **(2)**	30. **(2)**	60. **(2)**	90. **(1)**	30. **(4)**	60. **(2)**	90. **(1)**